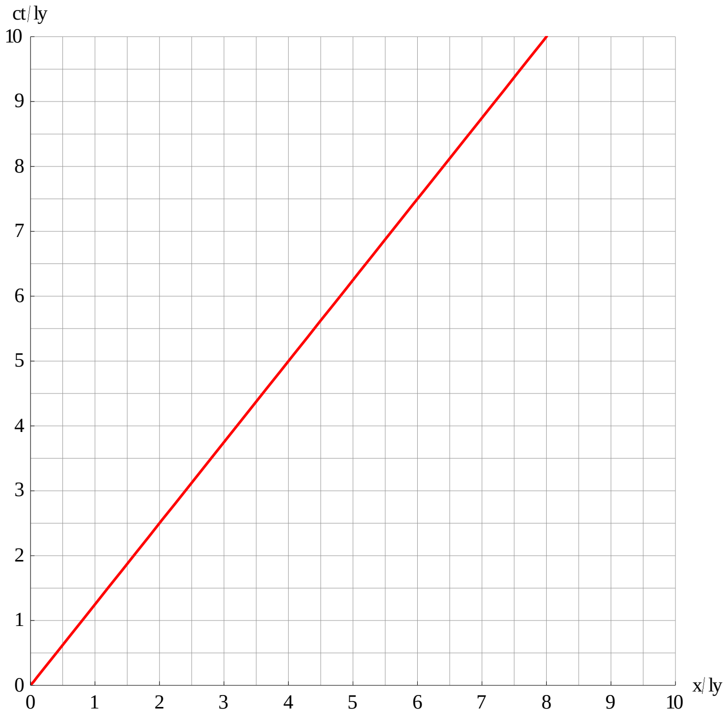
Teacher notes
Topic A

An instructive use of the invariant interval

The graph shows the worldline of a rocket on the spacetime axes of the Earth.

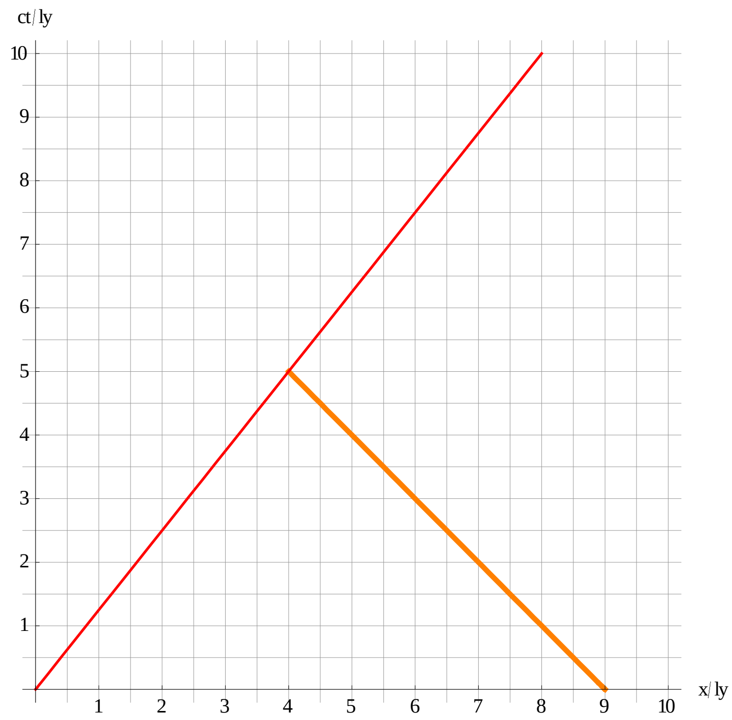


Find the speed of the rocket relative to the Earth.

At *ct* =0, a laser beam is directed towards the rocket from position *x* = 9.0 ly. When does the beam arrive at the rocket according to Earth and rocket observers?

The first part is easy: we see that a distance 4.0 ly is covered in a time given by *ct* = 5.0 ly, i.e. $t = \frac{5.0}{c}$ years and so $v = \frac{4.0}{\frac{5.0}{c}} = 0.8c$.

According to Earth, the distance between the rocket and the beam is decreasing at a rate (relative speed of the beam with respect to the rocket) of $1.8c$ and so the time taken is $\frac{9}{1.8c} = 5.0$ years i.e. $ct = 5.0$ ly according to Earth. This is shown clearly on the spacetime diagram below. (Note: do not be troubled by this higher than *c* speed: this is the rate, according to Earth, at which the distance between the rocket and the beam is decreasing. It is not the speed of an real, material object.)



What about the time for the rocket observer?

The arrival of the beam has coordinates $(x = 4.0, ct = 5.0)$ in the Earth frame and $(0, ct')$ in the rocket frame. The invariant interval gives

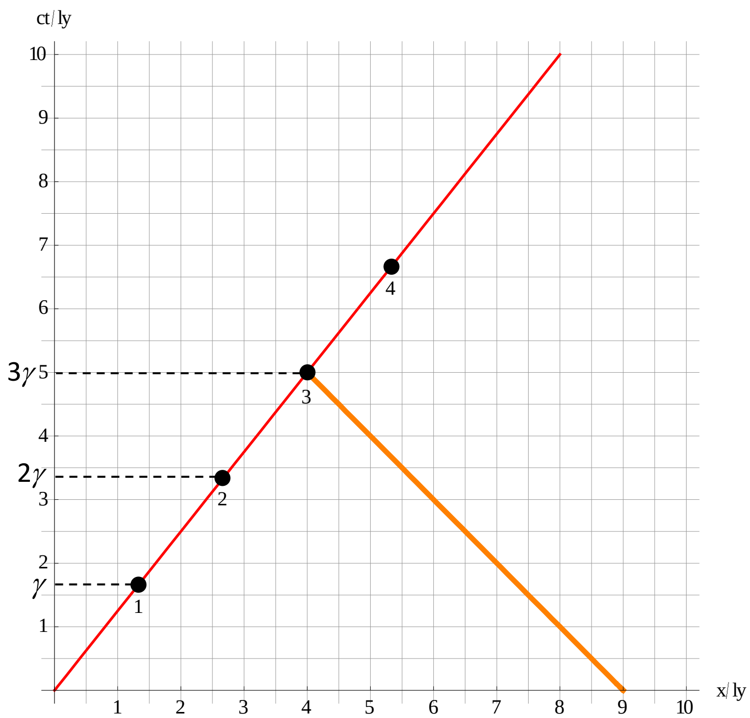
$$(ct')^2 - 0^2 = 5.0^2 - 4.0^2 = 9.0$$

i.e. $ct' = 3.0$ ly.

(Of course we could get the same result with a Lorentz transformation:

$$ct' = \gamma(ct - \frac{v}{c}x) = \frac{5}{3} \times (5.0 - 0.80 \times 4.0) = 3.0 \text{ ly}$$.)

By finding the scale on the rocket worldline we get the following diagram, which is consistent with what we found so far.


ct/ ly
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